AMRIT SCIENCE CAMPUS

B.Sc. CSIT Second Semester
Pre-Semester Examination - **Set "A"**

Subject: Mathematics - II | Full Marks: 60
Time: 3 hours | Pass Marks: 24

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A [$2 \times 10 = 20$].

Attempt ***any two*** *questions.*

1. Define linear independence and span of vectors in $\mathbb{R}^n$. Also define basis and dimension for a subspace of $\mathbb{R}^n$. For any matrix A with real entries, define null space and column space. Find a basis and dimension for the column space of $A = \begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & 1 & -1 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix}$. [1.5+1.5+1+6=10]

2. Define similar matrices and diagonalizable matrices. Let A and B be two matrices of order $n \times n$, with B being invertible. Prove that $B^{-1}A$ and AB^{-1} are similar matrices. Diagonalize the matrix $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$, if possible. [1+2+7=10]

3. Define orthonormal basis. Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{a}_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Let H be a subspace of $\mathbb{R}^4$ defined by $H = \text{span}\{\vec{a}_1, \vec{a}_2\}$. Does $\{\vec{a}_1, \vec{a}_2\}$ form a basis for H? If yes, then is it an orthonormal basis? Justify your answers. Define a matrix A by $A = [\vec{a}_1 \;\; \vec{a}_2]$. Explain why or why not a QR factorization of A is possible. If it is possible, then find a QR factorization of A. [1+1.5+0.5+7=10]

Group B [$8 \times 5 = 40$].

Attempt ***any eight*** *questions.*

4. Define consistency of a linear system. Given the system $x + y + z = 3$; $3x + 4y + 5z = 12$, check its consistency using Gaussian algorithm. [1+4=5]

5. Define lower and upper triangular matrices with an example for each of them. Find LU factorization of $\begin{bmatrix} 2 & 5 \\ 6 & -7 \end{bmatrix}$. [2+3=5]

6. Without expanding the determinant, evaluate $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 2 & 5 \\ 3 & 5 & 4 & 3 \\ 4 & 6 & 5 & 8 \end{vmatrix}$. [5]

7. How are linear transformation between two Euclidean spaces and its associated matrix defined? Is $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by $T(x, y) = (x, y, x+y)$ a linear transformation? If yes, then find its associated matrix. [1+3+1=5]

8. Define subspace of a vector space V over a field $\mathbb{F}$. Consider the vector space P_3 of polynomials with real coefficients of degree at most 3, over the real field. Let H be the subset of P_3 consisting of polynomials of the form $\{(x, ax + bx^3) : x \in \mathbb{R}\}$, where a and b are real numbers. Prove that H is a subspace of P_3. [1+4=5]

9. Find B-coordinate of the polynomial of the form $p(x) = 1 + 4x + 7x^2$ in P_2, where the given basis is $B = \{1 + x^2, x + x^2, (x + 1)^2\}$. [5]

10. State and prove the Pythagorean Identity for vectors in a Euclidean space. [5]

11. Define an Abelian Group with an example. Prove that $\langle \mathbb{R}^+, * \rangle$ is a group where $x * y = \dfrac{xy}{3}$ for all $x, y \in \mathbb{R}^+$. [1.5+3.5=5]

12. Define integral domain. Prove that $\langle \mathbb{Z}, +, \times \rangle$ is an integral domain. [2+3=5]

BEST OF LUCK